Worked Solutions

Edexcel C4 Paper L

1. $\int \frac{1}{y} \mathrm{d}y = \int 2x \ \mathrm{d}x$

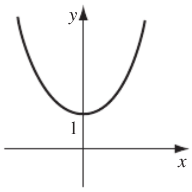
$\ln y = x^2 + c$

$y = \mathrm{e}, \quad x = 1: \qquad \ln \mathrm{e} = 1 + c$

$c = 0$

$\therefore \quad \ln y = x^2$

$y = \mathrm{e}^{x^2}$ **(4)**



(2)

---

2. (*a*) $V = \pi \int y^2 \ \mathrm{d}x = \pi \int_{-2}^{0} (t^2+1)^2 \ \frac{\mathrm{d}x}{\mathrm{d}t} \ \mathrm{d}t \qquad \frac{\mathrm{d}x}{\mathrm{d}t} = 1$ **(5)**

(*b*) $\pi \int_{-2}^{0} \left(t^2+1\right)^2 \ \mathrm{d}t = \pi \int_{-2}^{0} \left(t^4 + 2t^2 + 1\right) \mathrm{d}t$

$= \pi \left[\frac{t^5}{5} + \frac{2}{3}t^3 + t\right]_{-2}^{0}$

$= \pi \left[0 - \left(\frac{1}{5}(-32) + \frac{2}{3}(-8) + (-2)\right)\right] = \frac{206}{15}\pi$ **(2)**

---

3. (*a*) $(\pi, 0)$

(*b*) $\frac{\mathrm{d}y}{\mathrm{d}x} = x\cos x + \sin x$

when $x = 2.02, \quad \frac{\mathrm{d}y}{\mathrm{d}x} \approx 0.02$

when $x = 2.04, \quad \frac{\mathrm{d}y}{\mathrm{d}x} \approx -0.03$

(*c*) Area $= \int_0^{\pi} x\sin x \ \mathrm{d}x = \int_0^{\pi} x\frac{\mathrm{d}}{\mathrm{d}x}(-\cos x)\mathrm{d}x = \Big[-x\cos$

$= \Big[-x\cos x + \sin x\Big]_0^{\pi} = \pi$

---

4. (*a*) $17\left(\frac{17^2-1}{17^2}\right)^{\frac{1}{2}} = \frac{17}{17}\left[(17-1)(17+1)\right]^{\frac{1}{2}} = \sqrt{16}$

$= 4\sqrt{18} = 4\sqrt{9\times 2} = 12\sqrt{2}$

(*b*) $(1-x)^{\frac{1}{2}} = 1 + \frac{1}{2}(-x) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}(-x)^2 = 1 - \frac{1}{2}$

(*c*) put $x = \frac{1}{17^2}, \ \left(1 - \frac{1}{17^2}\right)^{\frac{1}{2}} \approx 1 - \frac{1}{2}\cdot\frac{1}{17^2} \approx \frac{577}{578}$

$\therefore \quad 17\left(\frac{577}{578}\right) \approx 12\sqrt{2}$

$\sqrt{2} \approx \frac{\cancel{17}}{12} \times \frac{577}{\cancel{578}_{34}}$

$\sqrt{2} \approx \frac{577}{408}$

---

5. (*a*) $\int x \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{k}\sin kx\right)\mathrm{d}x = \frac{x}{k}\sin kx - \frac{1}{k}\int \sin kx \;\mathrm{d}x$

$$= \frac{x}{k}\sin kx + \frac{1}{k^2}\cos kx + c \qquad \textbf{(4)}$$

(*b*) $\left[\frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x\right]_0^{\frac{\pi}{4}} = \frac{\pi}{8}\sin\frac{\pi}{2} + 0 - \left(0 + \frac{1}{4}\right) = \frac{1}{8}(\pi - 2)$ **(4)**

(*c*) $\cos 2x = 2\cos^2 x - 1$

$\therefore \quad 2\cos^2 x = \cos 2x + 1$

$$\int_0^{\frac{\pi}{4}} 2x\cos^2 x \;\mathrm{d}x = \int_0^{\frac{\pi}{4}} (x\cos 2x + x)\mathrm{d}x$$

$$= \frac{1}{8}(\pi - 2) + \left[\frac{x^2}{2}\right]_0^{\frac{\pi}{4}} = \frac{1}{8}(\pi - 2) + \frac{\pi^2}{32} \qquad \textbf{(4)}$$

---

6. (*a*) $\overrightarrow{AB} = \begin{pmatrix}3\\6\\12\end{pmatrix}$, line through A and B is $\mathbf{r} = \begin{pmatrix}1\\-1\\-5\end{pmatrix} + \lambda\begin{pmatrix}3\\6\\12\end{pmatrix}$ **(2)**

(*b*) If P lies on AB $\begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}1\\-1\\-5\end{pmatrix} + \lambda\begin{pmatrix}3\\6\\12\end{pmatrix}$

The equation is satisfied for x, y, z with $\lambda = \frac{1}{3}$ **(2)**

(*c*) $\overrightarrow{OP}\cdot\overrightarrow{AB} = \begin{pmatrix}2\\1\\-1\end{pmatrix}\cdot\begin{pmatrix}3\\6\\12\end{pmatrix} = 6 + 6 - 12 = 0$

$\therefore \quad OP$ is perpendicular to AB **(2)**

(*d*) $\overrightarrow{AP} = \begin{pmatrix}1\\2\\4\end{pmatrix}$

As $|\overrightarrow{OQ}| = |\overrightarrow{OA}|$ we have $\overrightarrow{AP} = \overrightarrow{PQ}$

$\therefore$ position vector of Q is $\begin{pmatrix}2\\1\\-1\end{pmatrix} + \begin{pmatrix}1\\2\\4\end{pmatrix}$ (1, -

$$= \begin{pmatrix}3\\3\\3\end{pmatrix}$$

---

7. (*a*) $\frac{\mathrm{d}y}{\mathrm{d}x} - 2x + x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$

$$\frac{\mathrm{d}y}{\mathrm{d}x}(1 + x) = 2x - y$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x - y}{1 + x}$$

(*b*) Gradient at $\left(1, \frac{9}{2}\right) = \frac{2 - \frac{9}{2}}{1 + 1} = -\frac{5}{4}$

(*c*) $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ where $y = 2x$ (from (a))

substitute $y = 2x$ into $y - x^2 + xy = 8$

$2x - x^2 + x(2x) = 8$

$x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

$x = -4, 2$

stationary points are $(-4, -8)$ and $(2, 4)$

---

8. (*a*) $\dfrac{dy}{dx} = \dfrac{(x+1)\cdot 1 - x \cdot 1}{(x+1)^2} = \dfrac{1}{(x+1)^2}$

at $x = 1$, $\dfrac{dy}{dx} = \dfrac{1}{4}$

$\therefore$ gradient of normal is -4

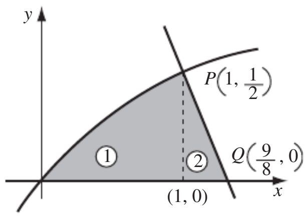
equation of normal is $y - \dfrac{1}{2} = -4(x-1)$

$y = -4x + 4\frac{1}{2}$ **(3)**

(*b*) normal cuts x-axis where $y = 0$

i.e. $4x = \dfrac{9}{2}$

$x = \dfrac{9}{8}$



shaded area = area (1) + area (2)

area (1) $= \displaystyle\int_0^1 \frac{x}{x+1}\,dx$

let $u = x + 1$

$\dfrac{du}{dx} = 1$

when $x = 1$, $u = 2$

$x = 0$, $u = 1$

$\therefore$ area (1) $= \displaystyle\int_1^2 \left(\frac{u-1}{u}\right) du = \int_1^2 \left(1 - \frac{1}{u}\right) du$

$= \Big[u - \ln u\Big]_1^2 = 2 - \ln 2 - (1 - \ln 1)$

$= 1 - \ln 2$

area (2) $= \frac{1}{2}$ base $\times$ height of $\triangle$

$= \dfrac{1}{2} \times \dfrac{1}{8} \times \dfrac{1}{2} = \dfrac{1}{32}$

$\therefore$ shaded area $= 1 - \ln 2 + \dfrac{1}{32}$

$= \approx 0.338$ units^2